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				5c. PROGRAM ELEMENT NUMBER 611102	
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				5e. TASK NUMBER	
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14. ABSTRACT During the period of this grant, there were many significant results on quantum adiabatic algorithms and quantum walks. On the adiabatic front, there were papers showing how to design error correcting codes specifically for these Hamiltonian based algorithms. There was also a paper showing in detail how poor choices in the design of the Hamiltonian controlling the adiabatic evolution could lead to algorithmic failure, but that these choices could be avoided. In general there was excellent progress in our understanding of the capabilities of the adiabatic algorithm.					
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a. REPORT UU	b. ABSTRACT UU	c. THIS PAGE UU			19b. TELEPHONE NUMBER 617-253-4871

## Report Title

Final Report for " Adiabatic Quantum Computing and Quantum Walks: Algorithms and Architectures"

### ABSTRACT

During the period of this grant, there were a many significant results on quantum adiabatic algorithms and quantum walks. On the adiabatic front, there were papers showing how to design error correcting codes specifically for these Hamiltonian based algorithms. There was also a paper showing in detail how poor choices in the design of the Hamiltonian controlling the adiabatic evolution could lead to algorithmic failure, but that these choices could be avoided. In general there was excellent progress in our understanding of the capabilities of the adiabatic algorithm. On the quantum walk front there was a breakthrough result demonstrating a quantum algorithm which can evaluate a NAND tree with fewer queries than the best possible classical algorithm. This result sparked a series of papers by other authors on this subject. In addition, during the course of the grant period, experimental evidence made it clear that quantum walks play an important role in biological systems, notably in photosynthesis.

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**List of papers submitted or published that acknowledge ARO support during this reporting period. List the papers, including journal references, in the following categories:**

**(a) Papers published in peer-reviewed journals (N/A for none)**

arXiv:0802.1874  
Title: Perturbative Gadgets at Arbitrary Orders  
Authors: Stephen P. Jordan, Edward Farhi  
Journal-ref: Phys. Rev. A 77, 062329 (2008)

arXiv:0712.1806  
Title: The Quantum Transverse Field Ising Model on an Infinite Tree from Matrix Product States  
Authors: Daniel Nagaj, Edward Farhi, Jeffrey Goldstone, Peter Shor, Igor Sylvester  
Journal-ref: Phys. Rev. B 77, 214431 (2008)

arXiv:quant-ph/0702144  
Title: A Quantum Algorithm for the Hamiltonian NAND Tree  
Authors: E. Farhi, J. Goldstone, S. Gutmann  
Journal-ref: Theory of Computing, vol 4, 169 (2008)

arXiv:quant-ph/0512170  
Title: Error correcting codes for adiabatic quantum computation  
Authors: Stephen P. Jordan, Edward Farhi, Peter W. Shor  
Journal-ref: Phys. Rev. A 74, 052322 (2006)

arXiv:quant-ph/0512159  
Title: How to Make the Quantum Adiabatic Algorithm Fail  
Authors: Edward Farhi, Jeffrey Goldstone, Sam Gutmann, Daniel Nagaj  
Journal-ref: International Journal of Quantum Information, Vol. 6, No. 3 (2008), 503-516

arXiv:0811.3171  
Title: Quantum algorithm for solving linear systems of equations  
Authors: Aram W. Harrow, Avinatan Hassidim, Seth Lloyd  
Journal-ref: Phys. Rev. Lett. vol. 15, no. 103, pp. 150502 (2009)

arXiv:0807.4994  
Title: Architectures for a quantum random access memory  
Authors: Vittorio Giovannetti, Seth Lloyd, Lorenzo Maccone  
Journal-ref: V. Giovannetti, S. Lloyd, L. Maccone, Phys. Rev. A 78, 052310 (2008)

arXiv:0807.0929  
Title: Environment-Assisted Quantum Transport  
Authors: Patrick Rebentrost, Masoud Mohseni, Ivan Kassal, Seth Lloyd, Alán Aspuru-Guzik  
Journal-ref: New J. Phys. 11, 033003 (2009)

arXiv:0807.0797  
Title: Landau-Zener Transitions in an Adiabatic Quantum Computer  
Authors: J. Johansson, M.H.S. Amin, A.J. Berkley, P. Bunyk, V. Choi, R. Harris, M.W. Johnson, T.M. Lanting, Seth Lloyd, G. Rose  
Journal-ref: Phys. Rev. B 80, 012507 (2009)

arXiv:0805.2757  
Title: Robustness of Adiabatic Quantum Computing  
Authors: Seth Lloyd

arXiv:0805.2741  
Title: Environment-Assisted Quantum Walks in Photosynthetic Energy Transfer  
Authors: Masoud Mohseni, Patrick Rebentrost, Seth Lloyd, Alán Aspuru-Guzik  
Journal-ref: Journal of Chemical Physics 129, 174106 (2008)

Number of Papers published in peer-reviewed journals: 10.00

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(b) Papers published in non-peer-reviewed journals or in conference proceedings (N/A for none)

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**(c) Presentations**

S. Lloyd, presentations at meetings:

September 2007, ``Quantum privacy,"  
Daejong University Physics Department Seminar, Korea.

September 2007, ``Quantum privacy,"  
NEC Conference on Quantum Information Processing, Princeton.

February 2008,  
``Quantum sensing and control,"  
AAAS 2008, Boston.

June 2008,  
``Quantum walks and photosynthesis,"  
Tokyo University physics department seminar, Tokyo, Japan.

July 2008,  
``Quantum computation and photosynthesis,"  
NTT Basic Research Laboratory seminar, Tokyo, Japan.

July 2008,  
``Adiabatic quantum computation,"  
three lectures delivered at Tokyo Institute of  
Technology, Tokyo, Japan.

July 2008,  
``Quantum vs. classical control,"  
keynote lecture, International Workshop on  
Quantum Control, Tokyo Institute of Technology, Tokyo,  
Japan.

September 2008,  
``Quantum computation and photosynthesis," keynote speech,  
DARPA QuBE meeting, Quantum Mechanics and Biology, Fairfax, VA.

November 2008,  
``Quantum algorithm for solving linear sets of equations,"  
MIT/Keck conference on difficult problems in quantum information theory,  
Cambridge, MA.

E. Farhi selected presentations at meetings

September 2008  
"Quantum scattering theory applied to games"  
QUINCE meeting Annapolis, Maryland

December 2008  
"A quantum computer can determine who wins a game faster than a classical computer"  
Brookhaven National Labs Colloquium

October 2007  
"A quantum computer can determine who wins a game faster than a classical computer"  
Caltech Colloquium

July 2007  
"A quantum algorithm for the Hamiltonian NAND tree"  
Computational Complexity Workshop, Leiden, Holland

May 2008  
"A quantum computer can determine who wins a game faster than a classical computer"  
MATHQCI 2008, Madrid, Spain

November 2005  
"Physics Based Approaches to Quantum Computing"  
City College of New York Colloquium

November 2007  
"A quantum computer can determine who wins a game faster than a classical computer"  
Princeton University Physics Colloquium

February 2008  
"A quantum computer can determine who wins a game faster than a classical computer"  
SQuInT meeting Santa Fe, New Mexico

Number of Presentations: 0.00

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Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts): 0

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Peer-Reviewed Conference Proceeding publications (other than abstracts):

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts): 0

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(d) Manuscripts

Number of Manuscripts: 0.00

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Patents Submitted

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Patents Awarded

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Awards

S. Lloyd became Fellow of American Physical Society, 2007

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Graduate Students

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
Stephen Jordan	0.13
Daniel Nagaj	0.13
<b>FTE Equivalent:</b>	<b>0.26</b>
<b>Total Number:</b>	<b>2</b>

#### Names of Post Doctorates

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
<b>FTE Equivalent:</b>	
<b>Total Number:</b>	

#### Names of Faculty Supported

<u>NAME</u>	<u>PERCENT SUPPORTED</u>	National Academy Member
Jeffrey Goldstone	0.08	No
Edward Farhi	0.21	No
Seth Lloyd	0.13	No
<b>FTE Equivalent:</b>	<b>0.42</b>	
<b>Total Number:</b>	<b>3</b>	

#### Names of Under Graduate students supported

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
<b>FTE Equivalent:</b>	
<b>Total Number:</b>	

#### Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: .....	1.00
The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:.....	1.00
The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:.....	1.00
Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):.....	1.00
Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:.....	0.00
The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense .....	0.00
The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields: .....	1.00

#### Names of Personnel receiving masters degrees

<u>NAME</u>
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<b>Total Number:</b>
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**Names of personnel receiving PhDs**

NAME

Stephen Jordan

Daniel Nagaj

**Total Number:**

2

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**Names of other research staff**

NAME

PERCENT SUPPORTED

**FTE Equivalent:**

**Total Number:**

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**Sub Contractors (DD882)**

**Inventions (DD882)**

**Scientific Progress**

The period of this grant was marked by many advances in both the areas of quantum adiabatic algorithms and in quantum walks. The objective was to understand the power of these methods and also to devise architectures for the implementation of these algorithms on a working quantum computer.

On the algorithm side, there was the paper "How to make the quantum adiabatic algorithm fail" which was written to show that an algorithm designer can make choices for the implementation of the quantum adiabatic algorithm which will doom the algorithm to fail even on simple problems. The point was to demonstrate that these choices can be avoided and that many of the examples in the literature of supposed algorithmic failure could indeed be avoided with better choices.

In the paper "Quantum Adiabatic Algorithms with Different Paths" it was shown that the adiabatic algorithm could be run repeatedly on the same instance of a combinatorial search problem. With each repetition, a different path in Hamiltonian space is chosen. This strategy can lead to algorithmic success in certain cases where the use of a single path leads to failure. This paper did away with a class of supposed counterexamples where it was argued that the adiabatic is doomed to fail.

In the paper "A Quantum Algorithm for the Hamiltonian NAND Tree" it was shown how to use quantum walk to get a speedup on the problem of evaluating a NAND tree at the root where the input 0's and 1's are on the leaves. The best possible classical algorithm for this task takes  $N^{.753}$  queries where  $N$  is the number of inputs. The quantum algorithm uses only  $\sqrt{N}$  queries and this is therefore provable quantum speedup. The techniques involve quantum scattering theory.

On the architecture side, it is important to develop for the adiabatic algorithm an error correction model. This is necessarily different than the error correction models used in conventional quantum computation. To this end Farhi, along with student Stephen Jordan and Professor Peter Shor developed a way of encoding qubits with more qubits in a way that produces a gap between the ground state where the computation is taking place and states which are bits flips away. This puts us on the road to devising an theory of fault tolerance for quantum adiabatic algorithms.

One of the purposes of this grant was to investigate the effects of noise on adiabatic quantum computation and on quantum walks. S. Lloyd, with graduate student William Kaminsky, developed models for calculating minimum energy gaps in adiabatic quantum computation due to domain formation. For quantum walks, they developed models for the effect of noise and decoherence on the propagation speed of the walk. They applied these methods to quantum walks performed by excitons in photosynthetic complexes.

They were able to develop a widely applicable theory of the effect of decoherence on quantum walks. In naturally occurring systems, decoherence can actually enhance the speed of transport in a quantum walk. The reason is that fully coherent quantum walks often suffer from Anderson localization, in which different paths in the walk interfere destructively, preventing long-range transport. Decoherence can in turn destroy the destructive interference, resulting in enhanced transport rates. They call this effect, Environment Assisted Quantum Transport (ENAQT). The use of environmental effects to enhance transport rates appears to be ubiquitous in energy transport in biological systems. Too much decoherence, by contrast, suppresses transport.

Lloyd and others also developed quantum-walk inspired designs and algorithms for quantum databases, including algorithms for quantum private queries, which allow one to access data in complete privacy. Lloyd and others developed a quantum algorithm for solving linear sets of equations in  $N$  variables. The algorithm runs in time  $O(\log N)$ , representing an exponential speedup over the best classical algorithm.

## **Technology Transfer**